Irrational Numbers

Any **number** that cannot be expressed in the form of p/q (where p and q are **integers** and q≠0.) is an **irrational number**. Examples √2, π, e and so on.

Its decimal expansion is non terminating and non-repeating.

2.20155424634895……., 3.14152455476441566787…..

1. Proof that root 2 is an irrational number.

Proof: Let us assume that √2 is a rational number.

So it can be expressed in the form p/q where p, q are co-prime integers and q≠0

$\sqrt{2} = p/q$

Here p and q are coprime numbers and q ≠ 0

Solving

$\sqrt{2} = p/q$

$\Rightarrow 2 = (p/q)^2$ On squaring both the side we get,

$\Rightarrow 2q^2 = p^2$……………………………(1)

$p^2/2 = q^2$

So 2 divides p and p is a multiple of 2.

$\Rightarrow p = 2m$

$\Rightarrow p^2 = 4m^2$ ……………………………….(2)

From equations (1) and (2), we get,

$2q^2 = 4m^2$

$\Rightarrow q^2 = 2m^2$

$\Rightarrow q^2$ is a multiple of 2

$\Rightarrow$ q is a multiple of 2

Hence, p and q have a common factor 2. This contradicts our assumption that they are co-primes. Therefore, p/q is not a rational number

√2 is an irrational number.

2. Prove that 3 + 2√5 is irrational

Proof: Let us assume that 3 + 2√5 is a rational number.

So, it can be written in the form a/b

3 + 2√5 = a/b

Here a and b are coprime numbers and b ≠ 0

Solving 3 + 2√5 = a/b we get,

=>2√5 = a/b – 3

=>2√5 = (a-3b)/b

=>√5 = (a-3b)/2b

This shows (a-3b)/2b is a rational number. But we know that √5 is an irrational number.

So, it contradicts our assumption. Our assumption of 3 + 2√5 is a rational number is incorrect.

Hence 3 + 2√5 is an irrational number **proved**

Do your self

Proof that these are irrational number.

a) $\sqrt{3}$,
b) $\sqrt{5}$,
c) $\sqrt{7}$,
d) $\sqrt{11}$,
e) $\sqrt{13}$,
f) $\sqrt{17}$,
g) $\sqrt{19}$
h) $5+2\sqrt{3}$,
i) $5-\sqrt{3}$,
j) $2\sqrt{7}$